Horizons in Cosmology

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Refs: Davis, T.M., and Lineweaver, C.H., Expanding Confusion: common misconceptions of cosmological horizons and the superluminal expansion of the universe, https://arxiv.org/abs/astro-ph/0310808; Melia, F., The apparent (gravitational) horizon in cosmology, *Am. J. Phys.*,**86** (8), Aug 2018, pgs. 585-593. https://arxiv.org/abs/1807.07587

Wholeness Chart in Words: Cosmological Horizons (in Billions of Light Years)

Type Horizon	Decelerating Universe	Accelerating Universe (assumes constant DE density)	Note	Value Now
Hubble sphere	L_H = radius at which galaxy recession velocity = c	As at left	Function of time but common meaning is now (our present time).	14.3
Particle horizon	L_P = distance to farthest light source we can see	As at left	Function of time, but common meaning is now This = observable universe	46
Cosmic event horizon	Not exist. Will eventually see every event.	L_E = distance from us now at which we will never see light emitted from there now Also = distance from us now at which light we emit now will never be seen by any observer there. Also = distance now to the farthest location we could ever reach if we left now at speed of light.	Function of time, but common meaning is now Acceleration moves distant space away faster than light from it can reach us.	16
Future visibility horizon	Not exist. Will eventually see every event.	L_F = farthest distance from us at which we will one day see light that has already been emitted	Function of time, but common meaning is now Galaxies beyond this we will never see.	61

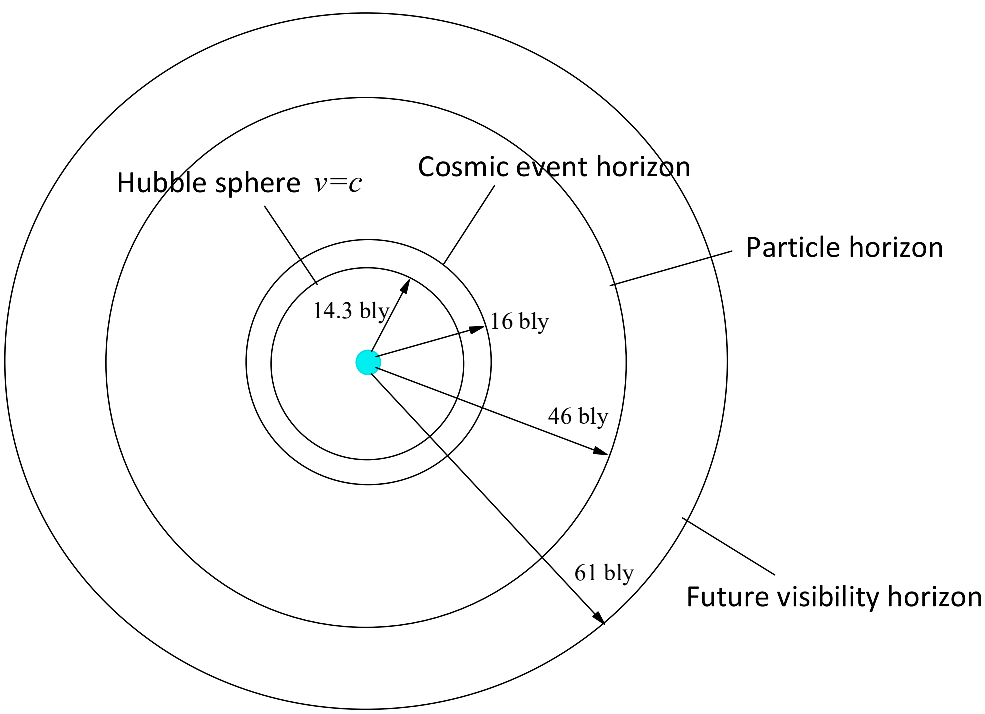

Figure 1. Cosmological Horizons Visually

Mathematics of Cosmological Horizons

1 The Universe's Metric and Physical Distances

The Friedmann–Lemaître–Robertson–Walker metric for a homogeneous, isotropic universe is

$$ds^2 = -c^2dt^2 + (a(t))^2 \left(d\chi^2 + S_k^2(\chi)d\psi^2\right) \tag{1}$$

where $a(t)$ is the expansion factor of the universe at time t, χ is the comoving radial coordinate where we can take Earth at $\chi = 0$, present time is t_0, $a(t_0) = 1$, and S_k depends on the curvature of the 3D spatial universe, i.e.

$$\text{Postitive curvature,}\;\; k = +1,\; S_k = \sin\chi; \;\; \text{Zero curvature,}\;\; k = 0,\; S_k = \chi; \;\; \text{Negative curvature,}\;\; k = -1,\; S_k = \sinh\chi\,. \tag{2}$$

The physical distance dL (measured with meter sticks) between two points fixed in the comoving coordinate system (such as two galaxies) at the same time (i.e., $dt = 0$), known as the proper distance, is

$$dL = ds = \sqrt{(a(t))^2\left(d\chi^2 + S_k^2(\chi)d\psi^2\right)}\,. \tag{3}$$

If we center our comoving coordinate system on the Earth and only concern ourselves with the radial (direct line) physical distance L to a galaxy with given comoving coordinate value χ, at time t, it is

$$L(t) = a(t)\chi\,. \tag{4}$$

2 The Hubble Sphere Distance

The Hubble "constant" (which changes in time as the universe evolves) is

$$\begin{gathered} H(t) = \frac{\dot{L}(t)}{L(t)} \qquad \left[\begin{array}{c}\text{physical velocity increase per unit physical distance of} \\ \text{galaxies (which are fixed in comoving coordinate system)}\end{array}\right]. \\ \text{Current measured value } H(t_0) \approx 70\frac{\text{km/sec}}{\text{megaparsec}} = 2.1\times 10^{-5}\text{ km/sec/light-year}. \end{gathered} \tag{5}$$

For L and its derivative using the same distance units (such as km), H has units of 1/sec.

At a distance where the galaxy recession velocity is the speed of light, the first row of (5) becomes

$$H(t) = \frac{c}{L(t)} = \frac{c}{L_H(t)} \quad\to\quad L_H(t) = \frac{c}{H(t)} \quad \xrightarrow[\text{time}]{\text{present}} \quad L_H(t_0) = \frac{c}{H(t_0)} = \frac{3\times 10^{-5}}{2.1\times 10^{-5}} = 14.3\text{ billion light-years}\,. \tag{6}$$

L_H, also called the Hubble distance D_H, is the radius of what is known as the Hubble sphere. Galaxies outside this sphere recede from us at a speed faster than light. This does not violate the special relativity postulate for maximum speeds in the universe, because it is spacetime itself that is expanding faster than the speed of light, not objects within that spacetime.

Alternative names for the Hubble distance are the gravitational horizon (typical in cosmology) or the apparent horizon (typical in other applications of general relativity). The "gravitation horizon" in black hole theory is the Schwarzschild radius, which separates regions from which light traveling toward the outside universe can reach the outside universe from regions where light cannot. The cause of the Schwarzschild radius is gravity, hence the name "gravitational horizon". In static black holes this radius/horizon does not change location. In our expanding, accelerating universe, however, it does.

The time dependence of the Hubble sphere varies with the model of the universe (different values for dark energy, matter, radiation densities). Depending on the model and t, its distance L_H could recede from us, or not. For the concordance model (our universe's current values for the densities) at t_0, the Hubble sphere is receding, i.e., L_H is increasing with time.

An aside on the Hubble constant

From the first row of (5) and (4), we have

$$H(t) = \frac{\dot{L}(t)}{L(t)} = \frac{\dot{a}(t)\chi}{a(t)\chi} = \frac{\dot{a}(t)}{a(t)}, \tag{7}$$

which is a very common way to express the Hubble constant.

End of aside

Different models of the universe lead to different $a(t)$, and thus from (7) different $H(t)$. This results, from (6), in different $L_H(t)$, i.e., different time evolution of the Hubble sphere.

3 The Particle Horizon

The particle horizon (also called the cosmological horizon, the comoving horizon, or the cosmic light horizon) is L_P distance away and represents the farthest reaches of the universe we can see today. The light from there has traveled for 13.8 billion years at local speed c, but because the universe was expanding while that light was traveling, the distance L_P it has traveled is more than 13.8 billion light-years.

But, during that entire travel time the light has been on a null spacetime path, i.e., $ds = 0$ all along the path. From (1), for a radial path towards us, this means

$$ds^2 = -c^2dt^2 + \left(a(t)\right)^2 d\chi^2 = 0 \qquad \rightarrow \qquad d\chi = c\frac{dt}{a(t)} . \tag{8}$$

To get the comoving coordinate distance traversed (which is not in meters or kilometers but simply a numerical value used for locations in the comoving system) at some time (in the history of the universe) t, we need to integrate (8) from the time of the Big Bang (or shortly thereafter when light could propagate through the universe), which we will designate as zero, to t.

$$\chi_P(t) = c\int_0^t \frac{dt'}{a(t')} \quad \xrightarrow[\text{time}]{\text{present}} \quad \chi_P(t_0) = c\int_0^{t_0} \frac{dt'}{a(t')} . \tag{9}$$

Using (4), we can then find the physical distance from us to the particle horizon at any time t in the history of the universe as

$$L_P(t) = a(t)\,\chi_P(t) , \tag{10}$$

and particularly, at the present time (where we don't carry out the actual calculations),

$$L_P(t_0) = a(t_0)\,\chi_P(t_0) \;\; = 46 \text{ billion light-years} . \tag{11}$$

Note that everything we see from 14.3 light-years away (Hubble sphere distance) to 46 billion light-years away is presenting receding from us at speeds greater than light.

4 The Cosmic Event Horizon

In an accelerating universe, light leaving now from far enough away can never reach us because it travels locally at speed c, but the 3D part of spacetime there is receding from us at greater than c, and that recession is accelerating. Relative to us, the light photons have velocity away from, not towards, us. This region of space is called the (cosmic) event horizon, and we herein label the physical distance to it as L_E.

Light leaving the event horizon now, the distance L_E beyond which we will never see light from, is, of course, light-like, i.e., $ds = 0$ all along its path. So, such light obeys (8). But instead of (9), we now need to integrate from present time t_0 to the end of the universe.

$$\text{Present time } t_0: \;\; \chi_E(t_0) = c\int_{t_0}^{t_{end}} \frac{dt'}{a(t')} \qquad \text{Any time } t \text{ in history of universe: } \;\; \chi_E(t) = c\int_{t}^{t_{end}} \frac{dt'}{a(t')} \tag{12}$$

$$\text{Concordance (accelerating) universe: } \; t_{end} = \infty$$

Using (4) again, as we did in (10), we find the physical distance from us to the event horizon at any time t in the history of the universe is

$$L_E(t) = a(t)\,\chi_E(t) , \tag{13}$$

and at the present time (where again we don't carry out the actual calculations)

$$L_E(t_0) = a(t_0)\,\chi_E(t_0) \;\; = 16 \text{ billion light-years} . \tag{14}$$

Note that L_E is the distance for each of the following.

1. The distance now at which we will never see light emitted from there now.
2. The distance now at which light we emit now will never be seen by any observer there.
3. The distance now to the farthest location we could ever reach if we left now at the speed of light.

5 A Note

One may sometimes hear the incorrect statement that light beyond the Hubble sphere can never reach us, since space beyond it is receding at greater than c. However, in the concordance model, as well as others, the Hubble sphere is expanding, so even though photons just beyond it are now traveling, relative to us, away from us, at some point in the future, the Hubble sphere will overtake those photons. When that happens, the photons will no longer be moving away from us, but toward us. And someday we would see them.

The event horizon, not the Hubble sphere, is the boundary point beyond which photons leaving now will never be seen by us. And as we have seen, at present time, the event horizon is larger than the Hubble sphere radius (16 billion light-years to 14.3). The math supports the conclusion of the prior paragraph.

6 Where Can We Go?

Note that the event horizon, beyond which we can never reach even traveling at the speed of light, is currently about 16 billion light-years from us. But the particle horizon, light from the furthest reaches of space that we can see, is about 46 billion light-years from us. Since 3D volume varies with the cube of the radius (of a 3-ball shaped region), that means we could actually ever reach only

$$\left(\frac{16}{46}\right)^3 = 4.2\% \tag{15}$$

of the visible universe. When we look out from Earth, almost 96% of what we see is forever inaccessible to us (excluding possibilities like Alcubierre warp drive).

7 The Future Visibility Horizon

There is one other horizon to consider, that beyond which we will never see any light, even that emitted a long, long time ago. The event horizon was for light emitted now, beyond which such light would never reach us.

But, light emitted in the past would already have travelled quite a distance toward us, and if such light, as of now, has passed closer to us than the event horizon, we will eventually see it. But, we can only go so far back into the past, to the Big Bang. Light emitted then, from very distant parts of the universe, could still be too far away for us to ever see it. But other light, emitted closer to us at the Big Bang, could still eventually reach us in the future.

There is a boundary beyond which light, even if emitted at the beginning of the universe, will not reach us. Inside that boundary, the very early light would reach us at some time (if not by now, then in the future). We call this boundary the <u>future visibility horizon</u>. Observers on Earth (assuming the Earth were to last forever) would, at some time, see light from objects inside this boundary, but never from those outside it. We find the coordinate distance χ_F and physical distance L_F to that boundary as in (16). Taking $t = t_0$ (now) gives us the physical distance to the future visibility horizon today.

$$\chi_F = c\int_0^\infty \frac{dt'}{a(t')} \qquad L_F(t) = a(t)\chi_F\,. \tag{16}$$

Wholeness Chart Math: Cosmological Horizons

χ = comoving radial coordinate L = physical radial distance H = Hubble constant

<u>Type Horizon</u>	<u>Decelerating Universe</u>	<u>Accelerating Universe</u>	<u>Note</u>
Hubble sphere	$L_H = \dfrac{c}{H(t)}$	As at left	$t = t_0$ for value now
Particle horizon	$\chi_P(t) = c\int_0^t \dfrac{dt'}{a(t')}$ $L_P(t) = a(t)\chi_P$	As at left	Common meaning $t = t_0$ (now)
Event horizon	No such thing. Light will eventually reach anywhere.	$\chi_E(t) = c\int_t^\infty \dfrac{dt'}{a(t')}$ $L_E(t) = a(t)\chi_E$	Common meaning $t = t_0$ (now)
Future visibility horizon	No such thing. Light will eventually reach anywhere.	$\chi_F = c\int_0^\infty \dfrac{dt'}{a(t')}$ $L_F(t) = a(t)\chi_F$	Common meaning $t = t_0$ (now)